change in the other elements. The optimal thrust program for changing n elements is the vector sum of the n programs for changing each element individually.

This important result has been utilized in several further papers. An analytical solution[4] for the long duration transfer between arbitrary coplanar or arbitrary coaxial ellipses was obtained by Krylov-Bogoliubov averaging.[5] This nonlinear solution contains some instructive examples of the occurrance of conjugate points on optimal trajectories.[4,6]

References

[1]Marinescu, A., "Optimal Low-Thrust Orbital Rendezvous," *Journal of Spacecraft and Rockets,* Vol. 13, July 1976, pp. 385-392.

[2]Gobetz, F.W., "Linear Theory of Optimum Low Thrust Rendezvous Trajectories," *Journal of the Astronautical Sciences,* Vol. 12, March 1965, pp. 69-76.

[3]Edelbaum, T.N., "Optimal Low-Thrust Rendezvous and Stationkeeping," *AIAA Journal,* Vol. 2, July 1964, pp. 1196-1201.

[4]Edelbaum, T.N., "Optimum Power-Limited Orbit Transfer in Strong Gravity Fields," *AIAA Journal,* Vol. 3, May 1965, pp. 921-925.

[5]Bogoliubov, N.N. and Mitropolsky, Y.A., *Asymptotic Methods in the Theory of Non-Linear Oscillations,* Hinduston Publishing Corp., Delhi, India, 1961.

[6]Edelbaum, T.N., "Optimization Problems in Powered Space Flight," *Recent Developments in Space Flight Mechanics,* AAS Science and Technology Series, Vol. 9, 1966.

Reply by Author to T.N. Edelbaum

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IN connection with T.N. Edelbaum's comment on my paper[1] the following points might clarify the matter.

Received Aug. 5, 1976; revision received Oct. 5, 1976.
Index category: Earth-Orbital Trajectories.
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1) T.N. Edelbaum's assertion concerning the solution for circular orbits given in Ref. 1 is not justified. Reference 2 published simultaneously with Ref. 3 is cited in order to show that Gobetz's variational problem differs from the variational problem in Refs. 1 and 3. The solution given in Ref. 1 completes the solution given by the author in Ref. 3 and one can speak of its equivalence with a solution given in Ref. 2 only on the basis of a comparative numerical application.

2) In Ref. 1 it is shown that the solution for elliptical orbits is valid both in the junction phase and in the terminal phase for maneuver durations compatible with the low thrust assumption.

The restriction for small changes of radius and true anomaly does not make incompatible the maneuver duration with low-thrust assumption as long as the magnitude of acceleration due to thrust does not exceed the imposed limits (at present $10^{-6} - 10^{-3}$g). With these values of acceleration due to thrust, the long maneuver durations are obvious in the interplanetary transfer or in the rendezvous on remote orbits. This is not the case treated in Ref. 1. The author's assumptions have permitted the easy integration of the equations of extremals without introducing substantial errors.

The use of the eccentric anomaly as an independent variable, recommended by T.N. Edelbaum, does not lead easily to rigorous analytical solutions both for optimal thrust program and for optimal trajectory. This can be clearly observed in Ref. 4.

References

[1]Marinescu, A., "Optimal Low-Thrust Orbital Rendezvous," *Journal of Spacecraft and Rockets,* Vol. 13, July 1976, pp. 385-392.

[2]Gobetz, F.W., "Linear Theory of Optimum Low Thrust Rendezvous Trajectories," *Journal of the Astronautical Sciences,* Vol. 12, March 1965, pp. 69-76.

[3]Marinescu, A., "Contributions a l'etude de certaines manoeuvres optimales pour le rendezvous des engines sur des orbites circulaires," *Proceedings of the XVIth International Astronautical Congress,* Athens, 1965, pp. 41-58.

[4]Tschauner, J., "Elliptic Orbit Rendezvous," *AIAA Journal,* Vol. 5, June 1967, pp. 1110-1113.